Far Eastern Mathematical Journal. 2023. V. 23. No 1. P. 106–111

UDC 517.95
MSC2020 35A07 + 35G30 + 35R30

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Boundary control problems for nonlinear reaction-diffusion-convection model

The solvability of the boundary control problem for a nonlinear model of mass transfer is proven in the case, when the reaction coefficient depends nonlinearly on concentration of substance and depends on spatial variables. The role of the control is played by the concentration value specified on the entire boundary of the domain.

Key words: *Nonlinear Mass-Transfer Model, Generalized Boussinesq Model, Reaction Coefficient, Boundary Control Problem.*

DOI: https://doi.org/10.47910/FEMJ202309

Introduction. Statement of boundary value problem

During a long period of time an interest in the study of boundary and extremum problems for heat and mass transfer equations has only increased (see, for example, [1–10]). Together with the search for the effective mechanisms for controlling physical fields in continuous media control problems have a number of other applications. Within the framework of the optimization approach these problems are reduced to some inverse problems (for the correctness of this approach, see [8, 11]).

In this paper we study the boundary control problem for the following mass transfer model considered in bounded domain $\Omega \subset \mathbb{R}^3$ with boundary Γ:

$$-\nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} + \nabla p = \mathbf{f} + \beta\mathbf{G}\varphi,\ \operatorname{div}\mathbf{u} = 0 \text{ in } \Omega, \tag{1}$$

$$-\operatorname{div}(\lambda(\mathbf{x})\nabla\varphi) + \mathbf{u}\cdot\nabla\varphi + k(\varphi,\mathbf{x})\varphi = f \text{ in } \Omega, \tag{2}$$

$$\mathbf{u} = \mathbf{0},\ \ \varphi = \psi \text{ on } \Gamma. \tag{3}$$

Here $\mathbf{u}$ is a velocity vector, function φ represents concentration of the pollutant, $p = P/\rho$, where P is pressure, $\rho = \text{const}$ is fluid density, $\nu = \text{const} > 0$ is constant kinematic viscosity, $\lambda = \lambda(\mathbf{x}) > 0$ is a diffusion coefficient, β is a coefficient of mass expansion, $\mathbf{G} = -(0,0,G)$ is acceleration of gravity, $\mathbf{f}$ and f are volume densities of external forces

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and external sources of substance, respectively. Finally, the function $k = k(\varphi, \mathbf{x})$ is a reaction coefficient, where $\mathbf{x} \in \Omega$. Below we will refer to the problem (1)–(3) for given functions $\mathbf{f}, f, \lambda, \beta, k$ and ψ as to Problem 1.

The global solvability of the Problem 1 and the local uniqueness of its solution are proven in [12]. The current article contains the solvability of the boundary control problem, in which the role of control is played by the function ψ from the boundary condition (3). Also we should bear in mind that the papers [9,13,14] are generalizing the Boussinesq approximation for various models, while the papers [15–17] are dedicated to the study a number of complicated hydrodynamic models.

1 Solvability of boundary value problem

Further we will use Sobolev functional spaces $H^s(D)$, $s \in \mathbb{R}$. Here D has the sense of either a domain Ω or of some subset $Q \subset \Omega$, or of the boundary Γ. By $\|\cdot\|_{s,Q}$, $|\cdot|_{s,Q}$ and by $(\cdot,\cdot)_{s,Q}$ we will denote the norm, the seminorm and the scalar product in $H^s(Q)$, respectively. $\|\cdot\|_Q$ and $(\cdot,\cdot)_Q$ stand for the norm and the scalar product in $L^2(Q)$, correspondingly, and $(\cdot,\cdot) \equiv (\cdot,\cdot)_\Omega$. Let us introduce $L^2_0(\Omega) = \{h \in L^2(\Omega) : (h, 1) = 0\}$, $V = \{\mathbf{v} \in H^1_0(\Omega)^3 : \operatorname{div} \mathbf{v} = 0 \text{ in } \Omega\}$, $L^p_+(D) = \{k \in L^p(D) : k \geq 0\}$, $p \geq 5/3$, and $L^\infty_{\lambda_0}(\Omega) = \{\lambda \in L^\infty(\Omega) : \lambda \geq \lambda_0 > 0\}$ and also present the product of spaces $H = H^1_0(\Omega)^3 \times H^1_0(\Omega)$, $W = V \times H^1_0(\Omega)$ and the functional space $H^* = H^{-1}(\Omega)^3 \times H^{-1}(\Omega)$ which is dual to H.

Let the following conditions hold:

(i) Ω is a bounded domain in $\mathbb{R}^3$ with a boundary $\Gamma \in C^{0,1}$;

(ii) $\lambda \in L^\infty_{\lambda_0}(\Omega)$, $\mathbf{f} \in L^2(\Omega)^3$, $f \in L^2(\Omega)$, $\mathbf{b} = \beta\mathbf{G} \in L^2(\Omega)^3$, $\psi \in H^{1/2}(\Gamma)$;

(iii) for any function $w \in H^1(\Omega)$ the embedding $k(w, \cdot) \in L^p_+(\Omega)$ is true for some $p \geq 5/3$, where p does not depend on w; and on any ball $B_r = \{w \in H^1(\Omega) : \|w\|_{1,\Omega} \leq r\}$ of radius r the following inequality takes place:

$$\|k(w_1, \cdot) - k(w_2, \cdot)\|_{L^p(\Omega)} \leq L\|w_1 - w_2\|_{L^4(\Omega)} \;\; \forall w_1, w_2 \in H^1(\Omega).$$

Here L is a constant which depends on r, but does not depend on $w_1, w_2 \in B_r$.

(iv) the nonlinearity $k(\varphi, \cdot)\varphi$ is monotone in the following sense: $(k(\varphi_1, \cdot)\varphi_1 - k(\varphi_2, \cdot)\varphi_2, \varphi_1 - \varphi_2) \geq 0 \; \forall \varphi_1, \varphi_2 \in H^1(\Omega)$.

(v) the function $k(\varphi, \cdot)$ is bounded in the sense that there exist a positive constants A_1, B_1 which depend on k, such that $\|k(\varphi, \cdot)\|_{L^p(\Omega_2)} \leq A_1\|\varphi\|^t_{1,\Omega} + B_1$, $p \geq 5/3$, $t \geq 0$.

The following lemmas hold (see [18]).

Lemma 1. *Under the conditions (i), $k_0 \in L^p_+(\Omega), p \geq 5/3$, $\mathbf{u} \in H^1(\Omega)^3$, $\operatorname{div} \mathbf{u} = 0$, $\mathbf{b} \in L^2(\Omega)^3$, $\lambda \in L^\infty_{\lambda_0}(\Omega)$, there exist positive constants $C_1, \delta_0, \delta_1, \gamma_1, \gamma_2, \gamma_2', \gamma_p, \beta_1$, which depend on Ω or on Ω and p, and there is also a constant β_0 which depends on $\|\mathbf{b}\|_\Omega$, such that the following relations are satisfied: $|(\mathbf{b}h, \mathbf{w})| \leq \beta_0\|h\|_{1,\Omega}\|\mathbf{w}\|_{1,\Omega}$ and*

$$|((\mathbf{w} \cdot \nabla)\mathbf{h}, \mathbf{z})| \leq \gamma_1\|\mathbf{w}\|_{1,\Omega}\|\mathbf{h}\|_{1,\Omega}\|\mathbf{z}\|_{1,\Omega} \, \forall \mathbf{w}, \mathbf{h}, \mathbf{z} \in H^1(\Omega)^3, \; h \in H^1(\Omega), \tag{4}$$

$$\sup_{\mathbf{v} \in H^1_0(\Omega)^3, \mathbf{v} \neq \mathbf{0}} -(\operatorname{div}\mathbf{v}, p)/\|\mathbf{v}\|_{1,\Omega} \geq \beta_1\|p\|_\Omega \;\; \forall p \in L^2_0(\Omega), \tag{5}$$

$$|(\lambda\nabla h, \nabla\eta)| \le C_1\|\lambda\|_{s,\Omega}\|h\|_{1,\Omega}\|\eta\|_{1,\Omega}, \qquad |(k_0 h, \eta)| \le \gamma_p\|k_0\|_{L^p(\Omega)}\|h\|_{1,\Omega}\|\eta\|_{1,\Omega},$$
$$|(\mathbf{u}\cdot\nabla h, \eta)| \le \gamma_2'\|\mathbf{u}\|_{L^4(\Omega)^3}\|h\|_{1,\Omega}\|\eta\|_{1,\Omega} \le \gamma_2\|\mathbf{u}\|_{1,\Omega}\|h\|_{1,\Omega}\|\eta\|_{1,\Omega} \quad \forall h,\eta \in H^1(\Omega),$$
$$\nu(\nabla\mathbf{v}, \nabla\mathbf{v}) \ge \nu_*\|\mathbf{v}\|^2_{1,\Omega},\ \forall\mathbf{v}\in H^1_0(\Omega)^3, \quad (\lambda\nabla h, \nabla h) \ge \lambda_*\|h\|^2_{1,\Omega}\ \forall h \in H^1_0(\Omega),$$

where $\nu_* = \nu\delta_0$ *and* $\lambda_* = \lambda\delta_1$.

Lemma 2. *Let the condition (i) hold. Then there exists a family of continuous non-decreasing functions* $M_\varepsilon : \mathbb{R}_+ \equiv [0,\infty) \to \mathbb{R}_+$, $M_\varepsilon(0) = 0$, *which depends on the parameter* $\varepsilon \in (0,1]$ *as well as on* Ω *and on* Γ, *such that for any non identically zero function* $\psi \in H^{1/2}(\Gamma)$ *there is a function* $\varphi_\varepsilon \in H^1(\Omega)$, *satisfying the conditions* $\varphi|_\Gamma = \psi$, $\|\varphi_\varepsilon\|_{L^4(\Omega)} \le \varepsilon$, $\|\varphi_\varepsilon\|_{1,\Omega} \le M_\varepsilon(\|\psi\|_{1/2,\Gamma})$ *for all* $\varepsilon \in (0,1]$.

Let us multiply the first equation in (1) by a function $\mathbf{v} \in H^1_0(\Omega)^3$, the equation (2) by a function $h \in H^1_0(\Omega)$ and integrate over Ω with the help of the Green's formulae. Thus, we are obtaining the weak formulation of the Problem 1:

$$\nu(\nabla\mathbf{u}, \nabla\mathbf{v}) + ((\mathbf{u}\cdot\nabla)\mathbf{u}, \mathbf{v}) - (p, \operatorname{div}\mathbf{v}) = (\mathbf{f}, \mathbf{v}) + (\mathbf{b}\varphi, \mathbf{v}) \ \ \forall\mathbf{v} \in H^1_0(\Omega)^3, \tag{6}$$
$$(\lambda\nabla\varphi, \nabla h) + (k(\varphi,\cdot)\varphi, h) + (\mathbf{u}\cdot\nabla\varphi, h) = (f, h) \ \ \forall h \in H^1_0(\Omega), \tag{7}$$
$$\operatorname{div}\mathbf{u} = 0 \text{ in } \Omega, \ \ \varphi = \psi \text{ on } \Gamma. \tag{8}$$

The triple $(\mathbf{u}, \varphi, p) \in H^1_0(\Omega)^3 \times H^1(\Omega) \times L^2_0(\Omega)$ which satisfies (6)–(8) will be called a weak solution of the Problem 1.

The following theorem takes place (see [12]).

Theorem 1. *Let us assume that the assumptions (i)–(v) hold. Then there exists a weak solution* $(\mathbf{u}, \varphi, p) \in H^1_0(\Omega)^3 \times H^1(\Omega) \times L^2_0(\Omega)$ *of the Problem 1 and the following estimates are true:*

$$\|\mathbf{u}\|_{1,\Omega} \le M_{\mathbf{u}} \equiv (\beta_0/(\nu_*\lambda_*))M_{f_1} + (1/\nu_*)\|\mathbf{f}\|_\Omega, \tag{9}$$
$$\|\varphi\|_{1,\Omega} \le M_\varphi \equiv (2/\lambda_*)(\gamma_2' M_{\mathbf{u}} + M_{f_1}) + M_\varepsilon(\|\psi\|_{1/2,\Gamma}), \quad \varepsilon = \nu_*\lambda_*/(2\beta_0\gamma_2'), \tag{10}$$
$$\|p\|_\Omega \le M_p \equiv \beta_2^{-1}[(\nu + \gamma_1 M_{\mathbf{u}})M_{\mathbf{u}} + \|\mathbf{f}\|_\Omega + \beta_0 M_\varphi], \ \beta_2 = \beta_1 - \delta, \quad \delta > 0, \tag{11}$$

where $M_{f_1} = \|f\|_\Omega + C_1\|\lambda\|_{L^\infty(\Omega)}M_\varepsilon(\|\psi\|_{1/2,\Gamma}) + \gamma_p(A_1 M^t_\varepsilon(\|\psi\|_{1/2,\Gamma}) + B_1)M_\varepsilon(\|\psi\|_{1/2,\Gamma})$.

If, besides, the condition $\mathrm{Re} + \mathrm{Ra} < 1$ *takes place, where* $\mathrm{Re} = (\gamma_1/\delta_0\nu)M_{\mathbf{u}}$ *and* $\mathrm{Ra} = (\gamma_2/\delta_0\nu)(\beta_0/\delta_1\lambda)M_\varphi$ *are dimensionless analogues of Reynolds number and of diffusion Rayleigh number (see [18, ch. 5]), then the weak solution of the Problem 1 is unique.*

2 Statement and solvability of control problem

In this section we will study a boundary control problem for the system (1)–(3), in which the role of the control is played by a boundary function ψ. We assume that ψ can be changed in a subset K, which satisfies the following condition:

(j) $K \subset H^{1/2}(\Gamma)$ and is a nonempty convex closed set.

Let us define functional spaces $X = H^1_0(\Omega)^3 \times H^1(\Omega) \times L^2_0(\Omega)$, $Y = H^{-1}(\Omega)^3 \times H^1(\Omega)^* \times L^2_0(\Omega) \times H^{1/2}(\Gamma)$ and set $\mathbf{x} = (\mathbf{u}, \varphi, p) \in X$ and introduce an operator $F = (F_1, F_2, F_3) : X \times K \to Y$ by formulae $\langle F_1(\mathbf{x}, \psi), (\mathbf{v}, h)\rangle = \nu(\nabla\mathbf{u}, \nabla\mathbf{v}) + (\lambda\nabla\varphi, \nabla h) + ((\mathbf{u}\cdot$

$\nabla)\mathbf{u}, \mathbf{v}) - (p, \operatorname{div} \mathbf{v}) + (k(\varphi, \cdot)\varphi, h) + (\mathbf{u} \cdot \nabla\varphi, h) - (\mathbf{f}, \mathbf{v}) - (\mathbf{b}\varphi, \mathbf{v}) - (f, h)$, $\langle F_2(\mathbf{x}, \psi), r\rangle = -(\operatorname{div} \mathbf{u}, r)$, $F_3(\mathbf{x}, \psi) = \varphi|_\Gamma - \psi \in H^{1/2}(\Gamma)$. Further we will rewrite a weak form (6)–(8) of the Problem 1 in the form of the operator equation $F(\mathbf{x}, \psi) = 0$.

Let $I : X \to \mathbb{R}$ be a weakly lower semicontinuous functional. Let us consider the following multiplicative control problem:

$$J(\mathbf{x}, \psi) \equiv (\mu_0/2)I(\mathbf{x}) + (\mu_1/2)\|\psi\|^2_{1/2,\Gamma} \to \inf,\ F(\mathbf{x}, \psi) = 0,\ (\mathbf{x}, \psi) \in X \times K. \quad (12)$$

The set of admissible pairs for the problem (12) is denoted by $Z_{ad} = \{(\mathbf{x}, \psi) \in X \times K : F(\mathbf{x}, \psi) = 0, J(\mathbf{x}, \psi) < \infty\}$. Let, in addition to (j), the following condition holds:

(jj) $\mu_0 > 0$, $\mu_1 \geq 0$ and K is a bounded set in $H^{1/2}(\Gamma)$ or $\mu_i > 0$, $i = 0, 1$ and the functional I is bounded from below.

We use the following cost functionals [18]:

$$I_1(\varphi) = \|\varphi - \varphi^d\|^2_Q,\ I_2(\varphi) = \|\varphi - \varphi^d\|^2_{1,Q},\ I_3(\mathbf{u}) = \|\mathbf{u} - \mathbf{u}^d\|^2_Q,\ I_4(p) = \|p - p^d\|^2_Q. \quad (13)$$

Here the function $\varphi^d \in L^2(Q)$ (or $\varphi^d \in H^1(Q)$) denotes a desired concentration field, which is given in a subdomain $Q \subset \Omega$. Functions $\mathbf{u}^d$ and p^d have a similar sense for either a velocity field or pressure.

Theorem 2. *Assume that the conditions (i)–(v) and (j), (jj) take place. Let $I : X \to \mathbb{R}$ be a weakly semicontinuous below functional and let $Z_{ad} \neq 0$. Then there is at least one solution $(\mathbf{x}, \psi) \in X \times K$ of the control problem* (12).

P r o o f. Let $(\mathbf{x}_m, \psi_m) = (\mathbf{u}_m, \varphi_m, p_m, \psi_m) \in Z_{ad}$ be a minimizing sequence for which the following is true: $\lim_{m\to\infty} J(\mathbf{x}_m, \psi_m) = \inf_{(\mathbf{x},\psi)\in Z_{ad}} J(\mathbf{x}, \psi) \equiv J^*$.

From the condition (jj) and from Theorem 1 it can be deduced that the following estimates hold:

$$\|\psi_m\|_{1/2,\Gamma} \leq c_1, \quad \|\mathbf{u}_m\|_{1,\Omega} \leq c_2, \quad \|\varphi_m\|_{1,\Omega} \leq c_3, \quad \|p_m\|_\Omega \leq c_4, \quad (14)$$

where the constants $c_1, c_2, \ldots$ don't depend on m. From the estimate (14) and from the condition (j) it follows that there exist weak limits $\psi^* \in K$, $\mathbf{u}^* \in H^1_0(\Omega)^3$, $\varphi^* \in H^1(\Omega)$, $p^* \in L^2_0(\Omega)$ of some subsequences of sequences $\{\psi_m\}$, $\{\mathbf{u}_m\}$, $\{\varphi_m\}$, $\{p_m\}$, respectively. With this in mind, it can be considered that, as $m \to \infty$, we have

$$\begin{gathered} \mathbf{u}_m \to \mathbf{u}^* \text{ weakly in } H^1(\Omega)^3 \text{ and strongly in } L^p(\Omega)^3, \quad p < 6, \\ \varphi_m \to \varphi^* \text{ weakly in } H^1(\Omega) \text{ and strongly in } L^s(\Omega), \quad s < 6, \\ p_m \to p^* \text{ weakly in } L^2(\Omega), \\ \psi_m \to \psi^* \text{ on } \Gamma \text{ weakly in } H^{1/2}(\Gamma) \text{ and strongly in } L^s(\Gamma), \quad s < 4. \end{gathered} \quad (15)$$

It is clear that $F_2(\mathbf{x}^*, \psi) = 0$, $F_3(\mathbf{x}^*) = 0$. Let us show that $F_1(\mathbf{x}^*, \psi^*) = 0$, i.e. that

$$\begin{gathered} \nu(\nabla\mathbf{u}^*, \nabla\mathbf{v}) + (\lambda\nabla\varphi^*, \nabla h) + ((\mathbf{u}^* \cdot \nabla)\mathbf{u}^*, \mathbf{v}) - (p^*, \operatorname{div} \mathbf{v}) + (k(\varphi^*, \cdot)\varphi^*, h) + \\ +(\mathbf{u}^* \cdot \nabla\varphi^*, h) = (\mathbf{f}, \mathbf{v}) + (\mathbf{b}\varphi^*, \mathbf{v}) + (f, h)\ \forall(\mathbf{v}, h) \in H. \end{gathered} \quad (16)$$

Let us also remind that $(\mathbf{x}_m, \psi_m)$ satisfies the relation

$$\nu(\nabla\mathbf{u}_m, \nabla\mathbf{v}) + (\lambda\nabla\varphi_m, \nabla h) + ((\mathbf{u}_m\cdot\nabla)\mathbf{u}_m, \mathbf{v}) - (p_m, \operatorname{div}\mathbf{v}) + (k(\varphi_m,\cdot)\varphi_m, h) + \\ +(\mathbf{u}_m\cdot\nabla\varphi_m, h) = (\mathbf{f}, \mathbf{v}) + (\mathbf{b}\varphi_m, \mathbf{v}) + (f, h)\ \forall(\mathbf{v}, h)\in H. \tag{17}$$

Let us pass to the limit in (17) as $m\to\infty$. From (15) it follows that all linear terms in (17) turn into corresponding ones in (16). Let us consider the nonlinear terms, starting with $(k(\varphi_m,\cdot)\varphi_m, h)$. From the condition (iii) it follows that $k(\varphi_m,\cdot)\to k(\varphi^*,\cdot)$ strongly in $L^{3/2}(\Omega)$ as $m\to\infty$. It is not difficult to show that from (15) a weak convergence $\varphi_m h\to\varphi^* h$ in $L^3(\Omega)$ for all $h\in H_0^1(\Omega)$ follows. Then $k(\varphi_m,\cdot)\varphi_m h\to k(\varphi^*,\cdot)\varphi^* h$ strongly in $L^1(\Omega)$ or $(k(\varphi_m,\cdot)\varphi_m, h)\to(k(\varphi^*,\cdot)\varphi^*, h)$ as $m\to\infty$ for all $h\in H_0^1(\Omega)$.

Arguing as in [9], we show that $((\mathbf{u}_m\cdot\nabla)\mathbf{u}_m, \mathbf{v})\to((\mathbf{u}^*\cdot\nabla)\mathbf{u}^*, \mathbf{v})$ as $m\to\infty$ for all $\mathbf{v}\in H_0^1(\Omega)^3$ and $(\mathbf{u}_m\cdot\nabla\varphi_m, h)\to(\mathbf{u}^*\cdot\nabla\varphi^*, h)$ as $m\to\infty$ for all $h\in H_0^1(\Omega)$.

As the functional J is weakly semicontinuous below on $X\times H^{1/2}(\Gamma)$, then from (14) it follows that $J(\mathbf{x}^*, \psi^*) = J^*$. □

Remark 1. It is clear that all cost functionals from (13) satisfy the conditions of the Theorem 2.

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Received by the editors
June 15, 2022

This work was supported by the Russian Science Foundation (Project No. 22-21-00271) and by the Ministry of Science and Higher Education of Russian Federation (Project No. 075-02-2023-946).

Сарицкая Ж. Ю., Бризицкий Р. В. Задачи граничного управления для нелинейной модели реакции – диффузии – конвекции. *Дальневосточный математический журнал.* 2023. Т. 23. № 1. С. 106–111.

АННОТАЦИЯ

Доказана разрешимость задачи граничного управления для нелинейной модели массопереноса в случае, когда коэффициент реакции нелинейно зависит от концентрации вещества, а также зависит от пространственных переменных. Роль управления играет значение концентрации, заданное на всей границе области.

Ключевые слова: *нелинейная модель массопереноса, обобщенная модель Буссинеска, коэффициент реакции, задача граничного управления.*